{ B.Sc Part II (Physics Hons) }
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Q What do you mean by optical path. State and explain Fermat's principle of extremum path and use it to the law of reflections and refraction of light?

Ans If a ray of light travels a distance 'd' in a medium of refractive index μ than the product μd is known as optical path in the medium

optical path = velocity of light in air x time of travel

Fermat's Principle :→

In 1655 Fermat state the principle of extremum path known after him. It is as follows if a ray of light travelling from one point to another by any number of reflections and refractions follows the particular path for which the time taken is least.

In many ~~no~~ number of cases the real path of light is the one for which the time taken is maximum rather than minimum. So in more general form " A ray of light travelling from one point to another by any number of reflections and refractions follows a path for which, compared with all other neighbouring paths, the time taken is either a minimum or max^{m} or Stationary"

Deduction of laws of reflection :—

Let a ray of light from a point A be reflected by a plane mirror ZZ' to another point B. Let 'i' and 'r' be the angle of

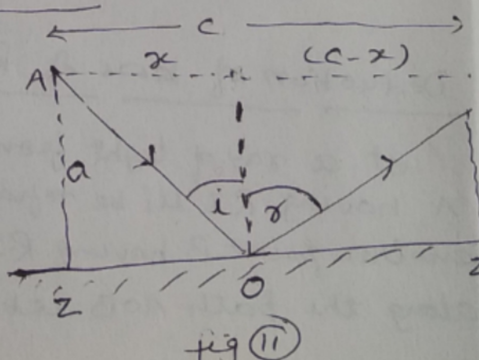

fig (II)

incident and reflection. Let 'a' & 'b' be the length of $\perp$ drawn from A and B on the mirror.

let $ZZ' = c$, $OZ = x$, $OZ' = (c-x)$

The path AOB is travelled in air so the optical path between A and B is

$$I = OA + OB = \sqrt{a^2+x^2} + \sqrt{b^2+(c-x)^2} \quad \text{——— (1)}$$

According to Fermat's principle O will have position such that the ~~po~~ optical path I is a min^m or max^m. So the first differential Co-efficient w.r.to x must be zero.

Now differentiating (1) w.r.to 'x' and equating the result to zero we get

$$\frac{dI}{dx} = \frac{1}{2}(a^2+x^2)^{-\frac{1}{2}}(2x) + \frac{1}{2}\left[(b^2+(c-x)^2\right]^{-\frac{1}{2}}(2)(c-x)(-1) = 0$$

This can be expressed as

$$\frac{x}{\sqrt{a^2+x^2}} = \frac{c-x}{\sqrt{b^2+(c-x)^2}} \quad \text{---- (2)}$$

from (1) figure

$$\frac{x}{\sqrt{a^2+x^2}} = \frac{ZO}{AO} = \sin i$$

and $$\frac{c-x}{\sqrt{[b^2 + (c-x)^2]}} = \frac{OZ'}{OB} = \sin r$$

from eq^n (2)

$\sin i = \sin r$

$\therefore i = r$

which is law of reflection.

Deduction of laws of Refraction :-

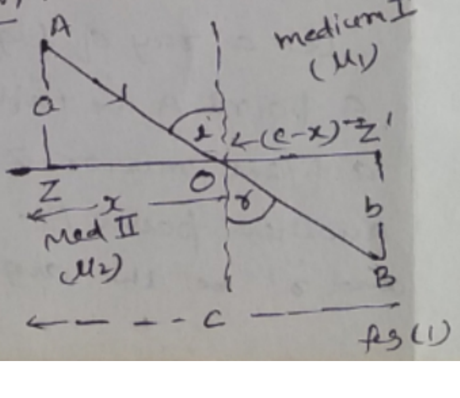

fig (1)

Let a ray of light from point A having R.I 'μ_1' be refracted to another point B having R.I 'μ_2'; along the path AOB. Let 'i' &

'r' be the angle of incidence and refraction. Let 'a'
and 'b' be the lengths of ⊥ drawn from A and B to refracting
surface.

Let $ZZ' = c$, $OZ = x$, $OZ' = (c-x)$

The optical path between A and B is given as

$$I = \mu_1 (AO) + \mu_2 (OB) \qquad (3)$$
$$= \mu_1 \sqrt{a^2+x^2} + \mu_2 \sqrt{b^2+(c-x)^2}$$

Diffrentiating eqn (3) w.r. to x and equating the result
to zero.

$$\frac{dI}{dx} = \mu_1 \left(\frac{1}{2}\right)(a^2+x^2)^{-\frac{1}{2}}(2x) + \mu_2 \left(\frac{1}{2}\right)\left[b^2+(c-x)^2\right]^{-1/2} \times (2)(c-x)(-1) = 0 \qquad (4)$$

This can be expressed as

$$\mu_1 \frac{x}{\sqrt{a^2+x^2}} = \mu_2 \frac{c-x}{\sqrt{b^2+(c-x)^2}}$$

from fig (11)

$$\frac{x}{\sqrt{a^2+x^2}} = \frac{OZ}{OA} = \sin i \quad \text{and}$$

$$\frac{c-x}{\sqrt{b^2+(c-x)^2}} = \frac{OZ'}{OB} = \sin r$$

$$\therefore \mu_1 \sin i = \mu_2 \sin r \quad (\text{from } (4))$$

$$\therefore \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}_1\mu_2$$

This is known as Snell's law of refraction.

This is law of refraction. Further it incident ray,
refracted ray and normal at the point of incidence
all lies in same plane.